M. Gopal
email: mgopal@bobcat.ent.ohiou.edu

W. P. Jepson
email: jepson@bobcat.ent.ohiou.edu

NSF, I/UCRC Corrosion in Multiphase Systems Center, Department of Chemical Engineering, Ohio University, Athens, OH 45701

The Study of Dynamic Slug Flow Characteristics Using Digital Image Analysis—Part II: Modeling Results

A definition is given for a Froude number in the liquid film ahead of the slug and it is seen that slug characteristics are strongly influenced by the Froude number. The mechanisms in the mixing zone of the slug are described in detail and are shown to be a function of the film Froude number. It is shown that the Hubbard and Dukler model for mixing length is inadequate. A new expression is proposed for the slug mixing length as a function of the film Froude number.

Introduction

Dukler and Hubbard (1975) published the first realistic mechanistic model for slug flow characteristics. From a mass balance over the film region and the slug, they established fundamental equations describing slug velocity. Nicholson et al. (1978) found that Eq. (1) was not adequate for prediction of the slug translational velocity, v_t. They defined a drift velocity, v_d, to model their results. Kouba (1986) concluded that the drift velocity, v_d, was significant for horizontal slug flow. Utilizing the theory of shearing flow over a wavy boundary developed by Benjamin (1968), and following Bendiksen (1984), he developed a complicated expression for the drift velocity as a function of pipe diameter. The slug translational velocity is then given by

$$v_t = (1 + C)v_s + v_d \quad (1)$$

The expression for the slug velocity, v_s, in Eq. (1) is found by summing the superficial liquid and gas velocity. The slug velocity, v_s, is also referred to as the mixture velocity, v_m.

Kouba (1986) also derived a generalized expression for the liquid film velocity ahead of the slug. By considering a mass balance between the slug and the liquid film, he was able to formulate a generalized expression for liquid film velocity ahead of the slug as follows:

$$v_{LF} = v_t\left[1 - \frac{(1-\alpha_s)}{(1-\alpha_{LF})}\right] + \frac{v_{LS}(1-\alpha_s)}{(1-\alpha_{LF})} \quad (2)$$

Equation (2) is the most generalized equation for the film velocity. It assumes stratified and completely segregated flow conditions between slugs and relates the film velocity to the velocity in the slug. At high slug velocities, the film between slugs is subject to large three-dimensional roll waves, which contribute to the liquid flow rate. With increasing slug frequency, video recordings reveal that the effect of these roll waves on the liquid film increases. In such situations, the contribution of both slugs and the film to the liquid flow rate becomes important and a liquid film velocity that is dependent on the slug frequency must be developed.

Jepson (1989) presented a physical model for the prediction of transition to slug flow. The model assumes that the slug is formed as a result of a hydraulic jump propagating along the conduit. He defined a dimensionless Froude number for the film ahead of the slug. The definition allowed a comparison between slugs and hydraulic jumps. Chow (1959) showed that the strength of the hydraulic jump is governed by the film Froude number, as shown in Fig. 1.

From a consideration of a momentum balance in the liquid and gas phase in the film ahead of the slug, the following equation is obtained:

$$\tau_{WG}\frac{S_G}{a_{GF}} - \tau_{WL}\frac{S_L}{a_{LF}} + \tau_i S_i\left(\frac{1}{a_{LF}} + \frac{1}{a_{GF}}\right) + (\rho_L - \rho_G)g\sin\theta = 0 \quad (3)$$

In their model, Taitel and Dukler (1976) assumed that the interface between gas and liquid was always smooth. The assumption of a smooth interface between gas and liquid is valid only at very low velocities, and this is not applicable to the slug flow regime. Andritsos and Hanratty (1987) using experimental data from 2.52 and 9.53-cm i.d. pipes, and liquid viscosities ranging from 1 to 70 cP, proposed a relationship between f_i and f_G. With this modification the model can adequately predict film heights in stratified flow.

For the work reported in this paper, flow visualization techniques were developed to conduct a detailed analysis of the local characteristics within the slugs. The flow was recorded on video and the images digitized on a computer. Digital image processing techniques to obtain data on slug translational velocity, liquid film height, and the length of the mixing region of the slug. Equations were then developed to predict the foregoing variables and the Froude number in the liquid film ahead of the slug. It is shown that the film Froude number strongly affects the slug flow characteristics.

Results

Slug Translational Velocity. Figure 2 shows the variation of the average ratio of v_t to v_s for both water and ARCO-PAK90™-carbon dioxide slug systems. These, as shown before, agree with those of Kouba (1986). A model for slug translational velocity, similar to that of Kouba (1986), is included in Fig. 2. The model uses Eq. (1) to predict the slug translational velocity.

Liquid Film Height. The film height is predicted by the model given by using Eq. (3), as per Taitel and Dukler. However, the presence of large-amplitude roll waves between slugs must be accounted for. Increasing slug frequency increases the

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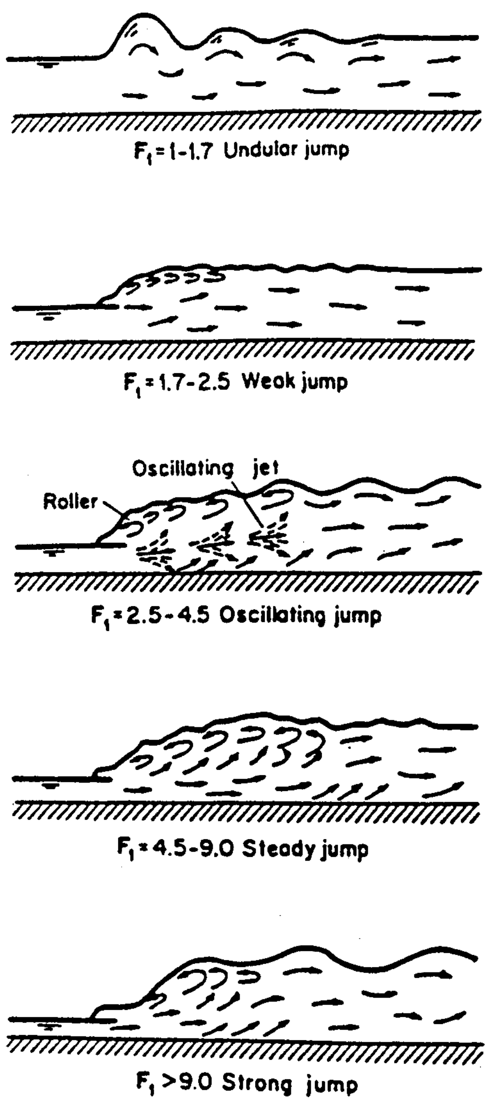

Fig. 1 Types of hydraulic jumps

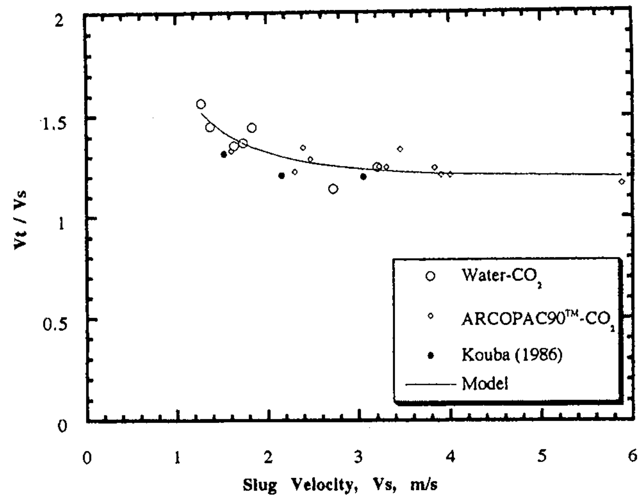

Fig. 2 Modeled variation of slug translational velocity with slug velocity

contributions of the roll waves to the film height. It becomes necessary to increase the ratio of the interfacial friction factor, f_i, to the gas phase friction factor, f_G, to higher values than those estimated by Andritsos and Hanratty (1987). Various ratios of f_i/f_G were used to predict the liquid film height in slug flow and it was found that values of this ratio greater than 50 did not alter the film height results. Hence, this value is used in this model.

In this study, the interfacial friction factor is estimated as a constant times the gas phase friction factor

$$f_i = k_1 f_G \tag{4}$$

where $k_1 = 50$.

Liquid Film Velocity. The liquid film velocity between slugs is not constant for slugs moving in a typical flowline. This is due to the drainage of liquid from the rear of the slug mixing with new incoming liquid. This rebuilds the liquid film to an equilibrium height on which the next slug can propagate. Further, at high gas velocities, waves are formed on the liquid film that affect the film velocity. Approximations are usually made to determine the liquid film velocity.

One model that can be used to predict the film velocity is the stratified model of Taitel and Dukler (1976). In slug flow, the total liquid flow rate is distributed between slugs and stratified liquid films between slugs. The stratified flow model neglects the contribution of the slugs to the total liquid flow rate and can overpredict the liquid film velocity. On the other hand, when the liquid flow rate is negligible compared with the gas flow rate, the slug characteristics are controlled by the gas, and the contribution from the liquid film velocity may be neglected. Using the generalized model of Kouba (1986), described by Eq. (2), we would expect a value somewhere in between the two extremes discussed in the foregoing. Crowley et al. (1988) showed that the liquid film velocity approached the superficial

Nomenclature

C = constant in slug translational velocity calculation constant in calculation of friction factor
D = hydraulic diameter pipe diameter
S = width of interface
f = friction factor
h = height
m = slope
v = velocity
Fr = Froude no.
c = intercept
g = acceleration due to gravity
l = length
a = phase area

Subscripts

EF = effective
GF = gaspocket ahead of slug
L = liquid
LF = liquid in film ahead of slug
WL = liquid-wall interface
c = center of pipe
f = liquid film
m = mixture, mixing
t = translational
G = gas
Go = refers to 1 atm
LS = liquid in slug
WG = gas-wall interface
b = bottom of pipe
d = drift
i = gas-liquid interface
s = slug
SG = superficial gas
α = void fraction
ρ = density
θ = angle of inclination
τ = wall shear stress

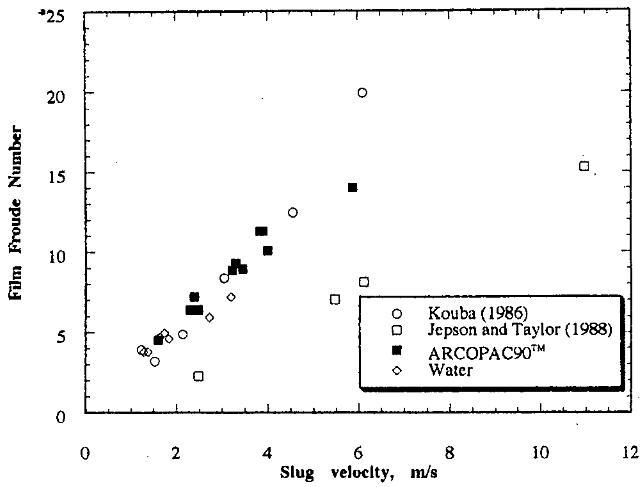

Fig. 3 Variation of film Froude number with slug velocity

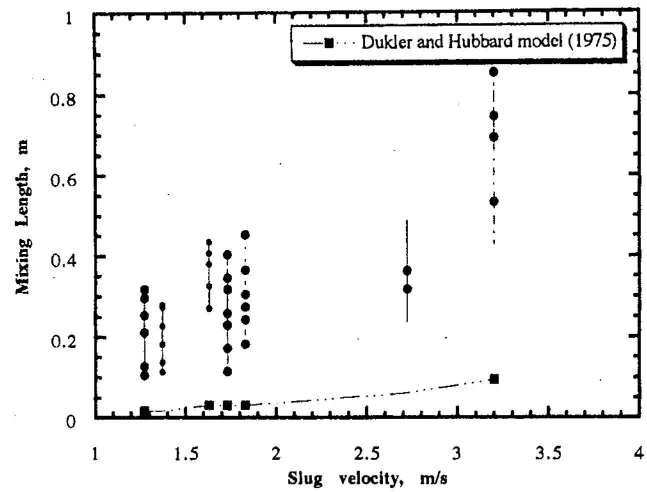

Fig. 4 Variation of mixing length with slug velocity water-CO_2 slug system

liquid velocity in slug flow. Therefore, an assumption can be made that, on average, the liquid film velocity, v_{lf}, will be close to the superficial liquid velocity. Hence, this model sets

$$v_{LF} \approx v_{SL} \tag{5}$$

This is used in the model for slug length prediction.

Film Froude Number. It has been shown that a slug is similar to a hydraulic jump (Jepson, 1989), and its strength can be determined by the Froude number in the liquid film ahead of the slug. The film Froude number is defined as follows:

$$\mathrm{Fr}_f = \frac{v_t - v_{LF}}{\sqrt{g h_{EF}}} \tag{6}$$

This definition of the film Froude number allows a comparison to be made between hydraulic jumps in open channel flow and slugs in pipelines. This results in slug characteristics being determined by the film Froude number.

Figure 3 shows the variation of the film Froude number as a function of the slug velocity for both water-carbon dioxide and ARCOPAK90™-carbon dioxide slug systems. It is seen that the lowest Froude number in the range studied is about 3.8. There is a linear increase in the film Froude number with an increase in slug velocity from 3.8 to a maximum of 14. The data for water as well as ARCOPAC90™ slugs fall in the same linear range of values. An increase of slug velocity from 1.5 to 3 m/s results in an increase of Froude number from 4 to 6. A further increase of slug velocity from 3 m/s to 6 m/s increases the Froude number from around 6 to 14. This is explained by the fact that the translational velocity of the slug increases in proportion to the slug velocity, but the height of the liquid film ahead of the slug does not change significantly. This causes the momentum term in the Froude number definition in Eq. (6) to increase significantly, while the gravity term in the denominator remains approximately the same. It is seen that the effect of viscosity on the film Froude number is negligible in this range of slug velocity.

Figure 3 also shows the Froude number calculations for the data of Kouba (1986) and Jepson and Taylor (1988). The data of Kouba is very similar to the data in the present study for slug velocities from 1 to 5 m/s. This is expected since both studies were conducted in 7.6-cm pipes. At higher velocities, gas expansion, resulting in slug acceleration, may account for higher Froude numbers in the data of Kouba. The data for Jepson and Taylor (1988) exhibits a lower range of velocities for a similar Froude number range. Since their studies were conducted in a 30-cm pipe, for the same slug velocity the Froude number is expected to be lower. It is seen that the Froude number definition works well for all range of data.

Mixing Zone. Figure 4 shows the variation of the mixing length in the slug as a function of slug velocity, for water-carbon dioxide slug systems. In this study, the end of the mixing zone was determined by the minimum distance into the slug where the void profile could be described as having reached a quasi-steady state. Visually, this also corresponds to the end of the frothy, highly turbulent region of the slug in the video image.

Figure 4 also shows the mixing length predicted by Dukler and Hubbard (1975). It is seen that the model does not predict the length of the mixing zone. The model was developed in terms of velocity heads to explain the assimilation of the liquid film into the slug. The length of the mixing zone is determined by the mixing of gas and liquid and not the liquid alone.

Jepson (1987) has shown that this length is proportional to the film Froude number ahead of the slug. Hence, knowing the film Froude number, the mixing length of the slug can be estimated. In this study, the mixing length of the slug has been modeled as an empirical function of the film Froude number. The variation in the mixing length was due to the large-scale turbulence and mixing in this region of the slug. It was seen that the mixing length increased with increasing slug velocities. This can be correlated to an increase in the film Froude number and the length of the mixing zone expressed as a function of the film Froude number. The dependence is given by

$$l_m = m . \mathrm{Fr}_f + c \tag{7}$$

where l_m = the length of the mixing zone, and m, c = linear regression coefficients.

Figure 5 shows a plot of mixing length in the slug as a function of film Froude number. From Eq. (7), the values of the regression coefficients are found to be $m = 0.13$, and $c = -0.31$. This gives the length of the mixing zone in meters. It was found that a correlation in terms of pipe diameter overpredicts the mixing length. Therefore, the correlation is given in terms of meters. This can be divided by pipe diameter and scaled for use in larger diameter pipes.

Equation (7) implies that below a film Froude number of 2.5, the mixing length will be zero. This is approximately true, since a Froude number of at least 2–2.5 is required for slugs to exist. At a film Froude number of about 4.0, the mixing length is about two times the pipe diameter. However, at about a Froude number of 8, the mixing length becomes ten times the pipe diameter. Using this definition implies that at very high

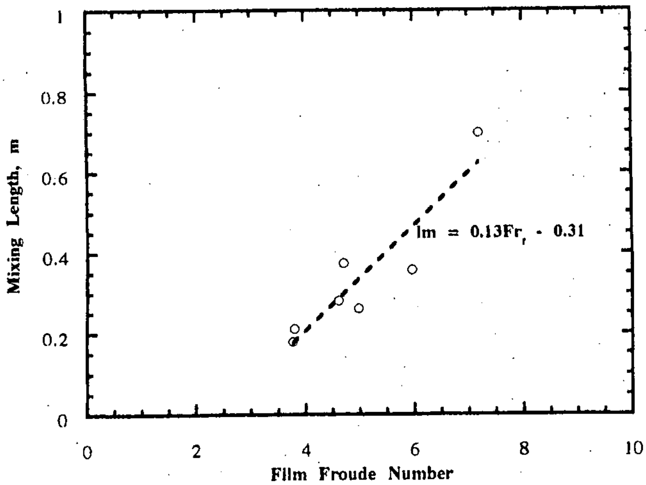

Fig. 5 Variation of slug mixing length with film Froude number

Froude numbers, the slug is almost all mixing zone, resulting in high gas entrainment and associated turbulence. This mechanism also explains the transition to pseudo-slug flow, and finally to annular flow.

Conclusions

A plot of v_t versus v_s was used to produce a model for the drift velocity was produced, and this was found to be a function of the slug velocity.

A model was developed to predict the liquid film height. Knowing the superficial gas and liquid velocities, the fluid properties, and the pipe diameter, the liquid film height ahead of the slug was predicted. A film Froude number was then calculated, which allowed the slug characteristics to be determined.

A new definition was developed for the mixing length. This was defined as the distance into the slug where the void profile became steady. It was seen from video images that this also corresponded to the end of the frothy, turbulent part of the slug. The mixing length was estimated as a linear function of the film Froude number ahead of the slug and was shown to vary from two to ten pipe diameters. It was found that the model given by Dukler and Hubbard (1975) generally underpredicted the length of the mixing zone.

References

Andritsos, N., and Hanratty, T. J., 1987, "Influence of Interfacial Waves in Stratified Gas-Liquid Flows," *AIChE Journal*, Vol. 33, p. 444.

Bendiksen, K. H., 1984, "An Experimental Investigation of the Motion of Long Bubbles in Inclined Tubes," *International Journal of Multiphase Flow*, Vol. 10, pp. 467–483.

Chow, V. Te., 1959, *Open-Channel Hydraulics*, McGraw-Hill, New York, NY.

Dukler, A. E., and Hubbard, M. G., 1975, "A Model for Gas-Liquid Slug Flow in Horizontal and Near Horizontal Tubes," *Industrial Engineering Chemistry Fundamentals*, Vol. 14, No. 4, pp. 337–347.

Jepson, W. P., 1987, "The Flow Characteristics in Horizontal Slug Flow," *3rd International Conference on Multiphase Flow*, Paper F2, p. 187.

Jepson, W. P., 1989, "Modeling the Transition to Slug Flow in Horizontal Conduit," *Canadian Journal of Chemical Engineering*, Vol. 67, p. 731.

Jepson, W. P., and Taylor, R. E., 1988, "Slug Flow and its Transitions in Large Diameter Horizontal Pipes," AERE-R12992, Harwell Laboratories, U.K.

Kouba, G. E., 1984, "Horizontal Slug Flow Modeling and Metering," Ph.D. thesis, University of Tulsa, Tulsa, OK.

Nicholson, M. K., Aziz, K., and Gregory, G. A., 1978, "Intermittent Two Phase Flow in Horizontal Pipes: Predictive Models," *Canadian Journal of Chemical Engineering*, Vol. 56, p. 653.

Taitel, Y., and Dukler, A. E., 1976, "A Model for Predicting Flow Regime Transitions in Horizontal and Near Horizontal Gas-Liquid Flow," *AIChE Journal*, Vol. 22, No. 1, p. 47.